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# EXPERIMENTALLY DETERMINED PENDULUM ANALOGY OF LIQUID SLOSHING IN SPHERICAL AND OBLATE-SPHEROIDAL TANKS

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## SUMMARY

An experimental investigation was conducted to determine the general liquid-sloshing characteristics (fundamental frequencies, horizontal or side slosh forces, and damping ratios) as well as quantities for a pendulum analogy that would effectively represent the fundamental mode of liquid sloshing in unbaffled oblate-spheroidal and spherical tanks over a range of liquid depths. Tanks having a diameter of 32.0 inches were used. Vertical and horizontal slosh forces were measured to determine several of the pendulum-analogy parameters. These parameters included (1) the pendulum mass, (2) the length of the pendulum arm, (3) the hinge-point location of the pendulum arm, (4) the maximum angles through which the pendulum can oscillate, and (5) the fixed mass.

The experimental results are presented in terms of dimensionless parameters that are, in general, independent of tank size, imposed longitudinal acceleration, and density and viscosity of the contained liquid. The experimentally determined fundamental-frequency and pendulum-analogy parameters are compared, whenever possible, with previously obtained analytical results.

## INTRODUCTION

Propellant sloshing is a potential source of disturbance critical to the stability and/or the structural integrity of space vehicles containing relatively large masses of liquid propellants, particularly if the propellant oscillations are coupled with either the attitude-stabilization-control frequency or the fundamental bodybending frequency of the vehicle. Considerable difficulty is found, however, when an attempt is made to include the hydrodynamic equations of liquid motion in a computer simulation of the vehicle motion for a stability analysis. Often, the solutions of the hydrodynamic equations do not exist in a form that is readily adapted to computer computation. Therefore, it is desirable to represent propellant sloshing as a mechanical analogy (e.g., either a pendulum or a spring-mass system) for which the linear, second-order differential equations of motion are readily adapted to digital or analog computer simulations.

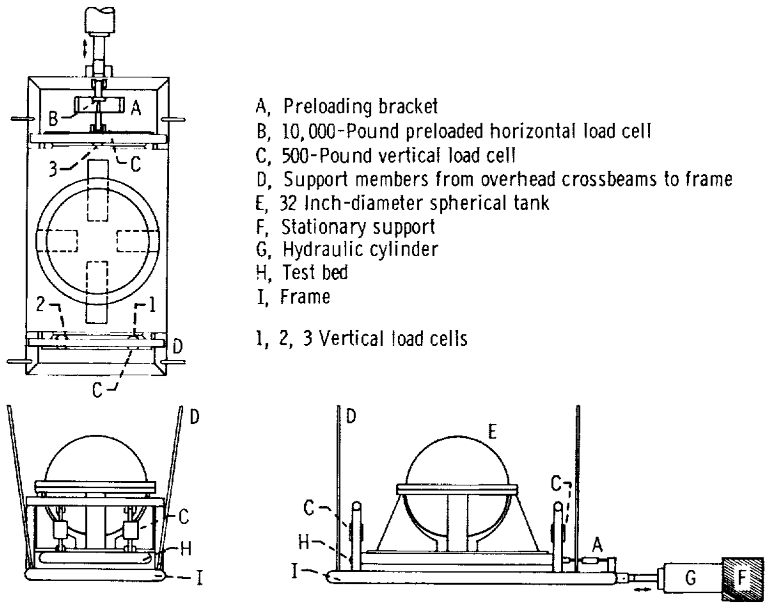
A pendulum model used to simulate effectively the fundamental mode of liquid sloshing generally consists of a fictitious pendulum mass suspended from a given point in each propellant tank to represent the sloshing liquid mass and a fictitious fixed mass in each tank to represent the nonsloshing mass. Generally, it is necessary to simulate only the fundamental mode of liquid sloshing in the mechanical analogy since the higher mode natural frequencies are normally much greater than the attitude-control frequencies and since the side forces produced at the higher modes are small. The quantities needed to describe the pendulum model of liquid sloshing are (1) the pendulum mass, (2) the length of the pendulum arm, (3) the hinge-point location of the pendulum arm, (4) the maximum angles through which the pendulum can oscillate, (5) the fixed mass, and (6) the centroid location of the fixed mass. Analytically determined values of quantities (1), (2), (3), and (6) are presented in reference 1 for oblate-spheroidal tanks. There have been no known investigations initiated, however, to determine experimentally the values of these quantities. Therefore, an experimental investigation was conducted at the NASA Lewis Research Center to determine the general liquid-sloshing characteristics (fundamental frequencies, horizontal or side slosh forces, and damping ratios) as well as the six previously mentioned quantities necessary to represent effectively the fundamental mode of liquid sloshing as a pendulum analogy.

The two tank configurations investigated were (1) an oblate-spheroidal tank having a circular diameter of 32.0 inches and a minor axis of 22.86 inches and (2) a spherical tank having a diameter of 32.0 inches. Experimental data were obtained over a range of liquid-depth ratios and excitation amplitudes for each tank configuration. The contained liquid was water. The experimental results are presented in terms of dimensionless parameters. The pendulum-analogy parameters are compared, whenever possible, with analytical results presented in reference 1.

## APPARATUS AND INSTRUMENTATION

An oblate-spheroidal tank having a circular diameter of 32.0 inches and a minor axis of 22.86 inches and a spherical tank having a diameter of 32.0 inches were fabricated from clear plastic. The contained sloshing liquid was water in all cases.

The experimental test facility, which is nearly identical to that described in references 2 and 3, is shown in figure 1. Each of the two tank configurations was mounted on a test bed that was suspended from a frame through three vertically oriented load cells and one horizontally oriented load cell. The frame was suspended from overhead crossbeams and was free to oscillate in one direction in the horizontal plane. The driving force was provided by a hydraulic piston and cylinder actuated by an electrically controlled




(a) Schematic view.


(b) Pictorial view.

Figure 1. - Experimental slosh-force test facility.

servovalve. The excitation amplitude could be varied from 0 to 1 inch, and the excitation frequency could be varied from 0 to 20 cps. A sinusoidal excitation waveform was used for this investigation. The electric and hydraulic control circuits for the driving mechanism were designed to enable the oscillatory motion of the frame, test bed, and tank to be "quick-stopped" at a point of zero velocity during any given cycle of oscillation so that only the residual forces resulting from the liquid sloshing could be measured.

The horizontal load cell was a piezoelectric quartz crystal that had been preloaded in compression. The vertical load cells were semiconductor-type transducers. The residual horizontal and vertical forces resulting from the liquid sloshing were sensed by the horizontal and vertical load cells, respectively, and the signals were displayed as continuously recorded oscillograph traces. The maximum error in obtaining the slosh forces from the oscillograph traces was approximately ±2.5 percent of the actual value.

## PROCEDURE AND DATA REDUCTION

### General Liquid-Sloshing Characteristics

Each tank was oscillated sinusoidally at a preselected excitation frequency (0.66 to 1.93 cps) that encompassed the range of fundamental frequencies of liquid motion over the range of liquid depths investigated and at an arbitrarily selected amplitude (0.010 to 0.200 in.). At each liquid depth investigated, the excitation frequency chosen was equal to the fundamental frequency of oscillation of the contained liquid since this provided the maximum wave height of the liquid surface and, therefore, the maximum slosh force at a given excitation amplitude (refs. 3 and 4). These maximum slosh forces are hereinafter referred to as the first-mode slosh forces. The maximum wave height of the liquid surface was established (1) for a steady-state oscillation of the liquid surface at low liquid depths and (2) for the point where the relatively flat liquid surface began to break up and shower liquid throughout the tank and before any rotary or swirling motion of the liquid could take place at the higher liquid depths. When the fundamental or first natural-mode waveform had built up to its maximum height on the tank wall, the oscillatory motion of the tank was quick-stopped, and the residual slosh forces were recorded.

The fundamental frequencies of the liquid oscillations were determined for the first several slosh-force cycles occurring immediately after the quick stop and are presented in the form of the fundamental-frequency parameter $\eta_e = \omega_e \sqrt{r/g}$. (Symbols are defined in appendix A.) The values of the first-mode horizontal slosh forces were determined from the first force peak occurring immediately after the quick stop; the horizontal forces are presented in the form of the first-mode slosh force parameter $\lambda = F_s/\rho g \alpha D^3$. The first-mode damping ratios were calculated by averaging the first two successive values of the logarithmic decrement $\delta = \ln\left[(F_s)_n/(F_s)_{n+1}\right]$ on each oscillograph trace, where $(F_s)_n$ was the peak force on one slosh cycle and $(F_s)_{n+1}$ was the peak force on the succeeding cycle.

## Quantities for Pendulum Analogy

Pendulum mass. - In order to determine experimentally the pendulum mass (or effective sloshing mass), it was necessary to oscillate each tank at an excitation frequency much less than the fundamental frequency of the contained liquid such that, for steady-state conditions, the liquid would oscillate at exactly the excitation frequency. Excitation frequencies were varied from 40 to 70 percent of the fundamental frequencies over the range of liquid-depth ratios investigated, and excitation amplitudes were varied from 0.200 to 0.900 inch. Specific values of the excitation frequency and amplitude at each liquid-depth ratio were chosen such that the wave height of the liquid surface was as large as possible while the liquid surface remained relatively flat. Thus, the accuracy of measuring the relatively large slosh forces present was increased. Once the wave height reached a steady-state value, the oscillatory motion of the tank was quick-stopped, and the horizontal slosh forces were determined from the first force peak occurring immediately thereafter. The method of calculating the pendulum mass is presented in appendix B.

Hinge-point location. - The excitation frequency was equal to the fundamental frequency of the contained liquid. The excitation amplitude was varied from 0.005 to 0.050 inch depending on the liquid-depth ratio. The wave height was allowed to increase toward a maximum value with the following restrictions: (1) the liquid surface would remain relatively flat with no splashing of the contained liquid and (2) the liquid would be oscillating in exactly the same direction as the driving force (no swirl or rotary motion of the liquid surface) so that the slosh-force traces of vertical load cells 1 and 2 (fig. 1(a), p. 3) would be in phase and equal in magnitude. The oscillatory motion of the tank was then quick-stopped, and the residual forces resulting from the liquid sloshing were recorded.

A typical slosh-force oscillograph trace is shown in figure 2. The oscillograph trace labeled Tank position indicates (1) the initial oscillatory motion of the frame, test bed, and tank and (2) the point at which the oscillatory motion was quick-stopped. The data trace labeled $F_s$ indicates the slosh-force measurements obtained from the horizontal load cell, while the traces labeled $F_1$, $F_2$, and $F_3$ indicate the force measurements obtained from vertical load cells 1, 2, and 3, respectively. (The position of the vertical load cells is noted in fig. 1.) The horizontal and vertical slosh forces were determined from the first force peaks occurring immediately after the tank was quick-

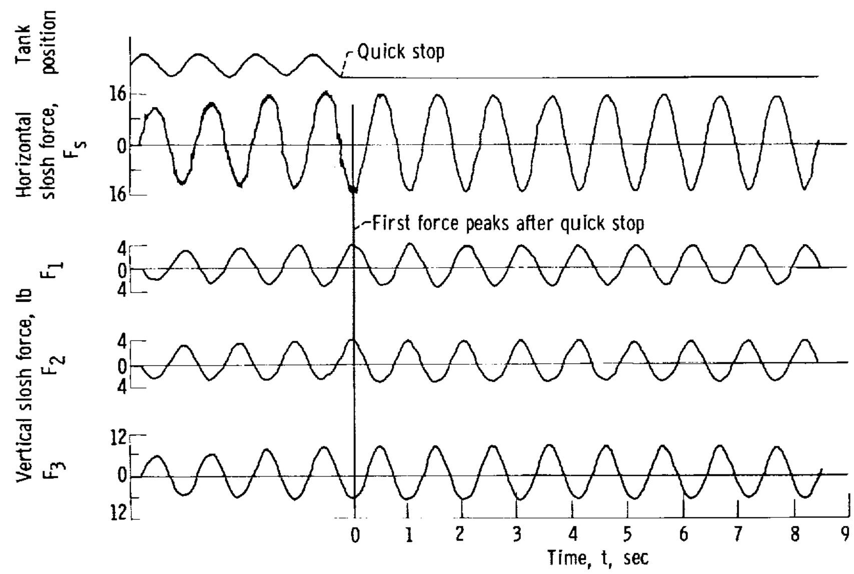



Figure 2. - Typical slosh-force oscillograph trace for spherical tank. Liquid-depth ratio, 0.50; excitation-amplitude parameter, 0.00094.

stopped, and the moment about the center of the tank was calculated. The method of obtaining the hinge-point location is given in appendix B.

Liquid-slosh angle. - The angle from the horizontal at which the liquid surface oscillated $\gamma_l$ (fig. 3) was measured visually by a protractor-type device when the liquid surface had attained its maximum height on the tank wall for different values of excitation-amplitude parameter and liquid-depth ratio. The accuracy of this device was better than $\pm 2^{\circ}$.

# EXPERIMENTAL RESULTS

## General Liquid-Sloshing Characteristics

Fundamental frequency. - By using an "equivalent cylindrical tank" method (ref. 5), the fundamental frequencies of oscillation of the liquid contained in the oblate-spheroidal and spherical tanks were calculated from the equation

$$\omega_c = \sigma \sqrt{\frac{g}{r} \epsilon \tanh\left(\frac{h_c}{r} \epsilon\right)} \tag{1}$$

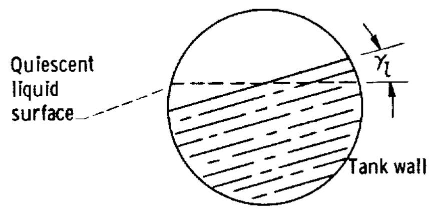



Figure 3. - Definition of liquid slosh angle.

The experimental values of the correction factor $\sigma$ obtained in references 2 and 5 to provide closer agreement between the calculated and experimental values of the

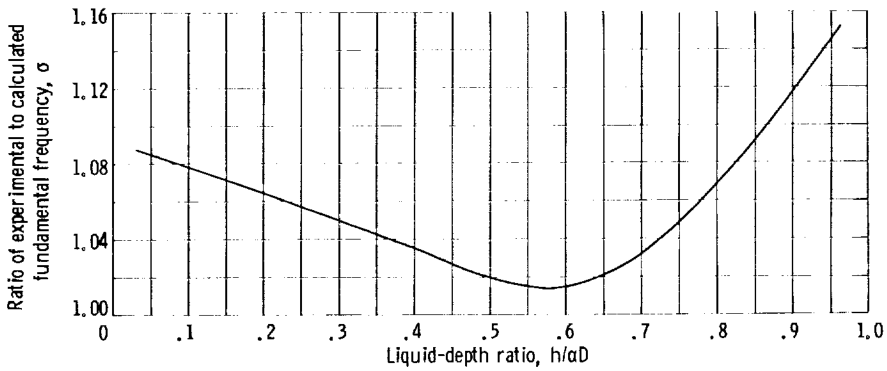



Figure 4. - Ratio of experimental to calculated fundamental frequency for oblate-spheroidal tank (refs. 2 and 5).

fundamental frequency over a wide range of liquid-depth ratios are shown in figure 4. The values of the fundamental frequency had also previously been obtained analytically from exact solutions of the hydrodynamic equations of liquid motion for a spherical tank (ref. 4). The analytical, calculated, and experimental values of the fundamental frequency are presented in terms of the parameters $\eta_a = \omega_a \sqrt{r/g}$, $\eta_c = \omega_c \sqrt{r/g}$, and $\eta_e = \omega_e \sqrt{r/g}$, respectively, in figure 5 for the oblate-spheroidal and spherical tanks. The fundamental-frequency parameter increased with liquid-depth ratio for the two tank configurations investigated. The analytical, calculated, and experimental values were in

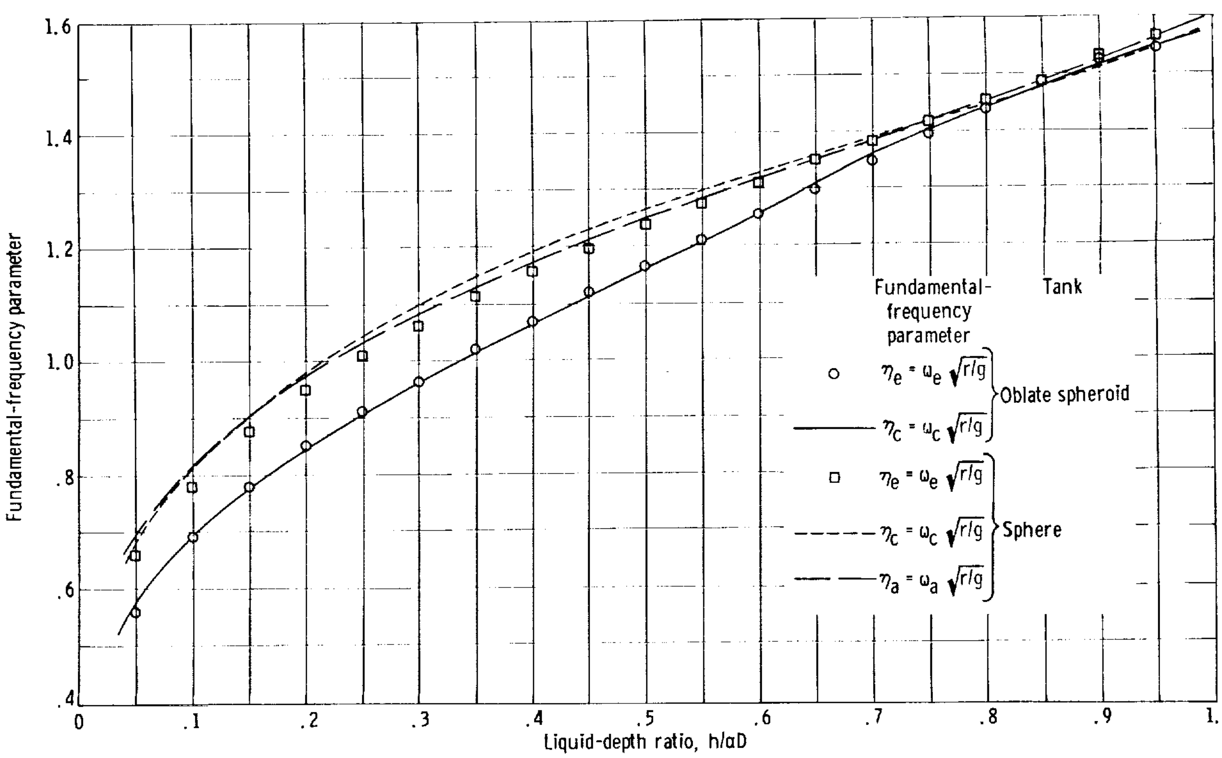



Figure 5. - Fundamental-frequency parameter.

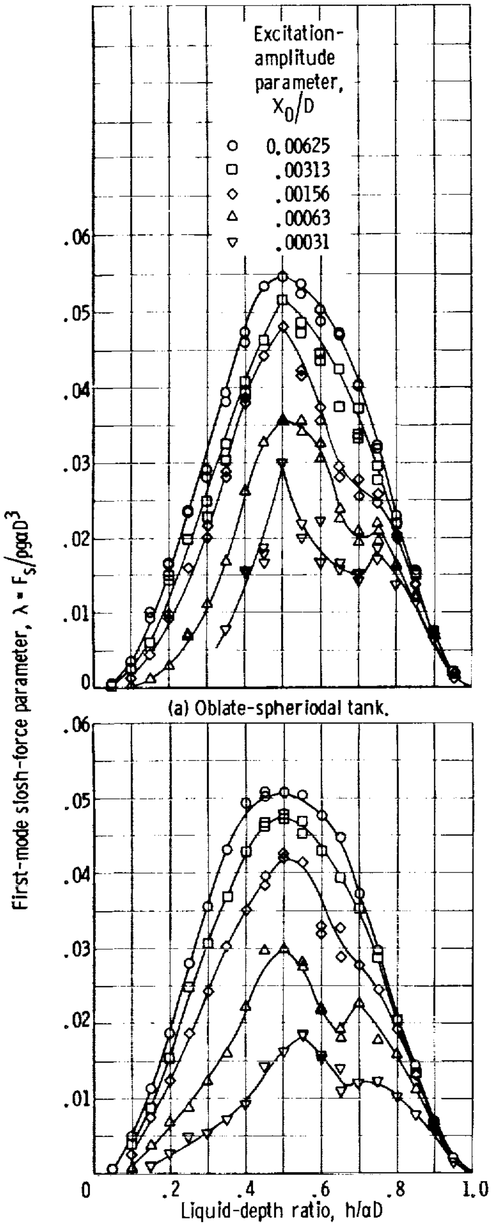


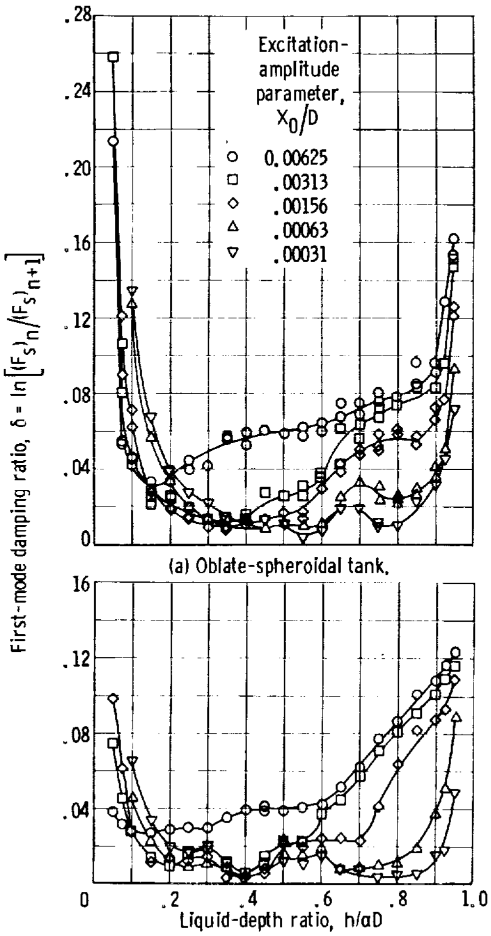

Figure 6. - First-mode slosh-force parameter as function of liquid-depth ratio.

close agreement, with less than 4 percent variation in all cases. Each data point shown represents an average of several values; the maximum deviation from the average was ±1.5 percent.

Horizontal slosh forces. - The horizontal slosh forces were obtained for an excitation frequency equal to the fundamental frequency of the contained liquid at each liquid-depth ratio investigated and are, therefore, the maximum slosh forces that could be obtained for a given excitation amplitude. These slosh forces are presented in figure 6 in terms of the first-mode slosh-force parameter $\lambda = F_S/\rho g \alpha D^3$ for the oblate-spheroidal tank and the spherical tank. The maximum slosh forces for a given excitation amplitude were found when each tank was at or very near a half-full condition. The slight dip that appeared in the faired curves at liquid-depth ratios of 0.60 to 0.75 and for excitation-amplitude parameters equal to or less than 0.00156 occurred because the liquid near the surface tended to separate from the tank wall as the wave height of the liquid surface approched the maximum height that could be obtained. To maintain a relatively flat liquid surface, it was necessary to quick-stop the oscillatory motion of the tank somewhat prematurely, which thereby reduced the slosh forces slightly.

Damping ratios. - The first-mode damping ratios obtained in the oblate-spheroidal and spherical tank configurations are presented in figure 7 for a range of excitation amplitudes. At low liquid-depth ratios, the damping ratios increased with a decrease in the excitation amplitude and liquid-depth ratio because of the small mass of liquid oscillating over a large tank wall area. At high liquid-depth ratios, the damping ratios increased with an increase in excitation amplitude and liquid-depth ratio because of (1) the curvature of the tank wall and (2) the small portion of liquid that splashed from the tank wall and tended to produce ripples on the surface of the liquid.

Scaling factors. - The fundamental frequencies of liquid oscillations in oblate-spheroidal or spherical tanks of any size under any imposed longitudinal acceleration can be calculated directly from equation (1) (p. 6), or from figure 5 for geometrically similar




(b) Spherical tank.

Figure 7. - First-mode damping ratio as function of liquid-depth ratio.

tank configurations. The fundamental frequencies are generally independent of the kinematic viscosity of the contained liquid (ref. 3).

When it is necessary to predict sloshing characteristics in a full-scale tank from experimental tests conducted in a scale-model tank, it is desirable for the values of the viscosity parameter $\nu/\sqrt{gD^3}$ for the two tanks to be nearly the same. But in cases where the full-scale tank is appreciably larger than the model tank, it is generally impossible or impractical to obtain a sloshing liquid with a low enough kinematic viscosity for the model tank. The slosh-force parameter is virtually independent of tank size and liquid viscosity, however, when the tank diameter is large or when the liquid viscosity is small (values of the viscosity parameter less than $10^{-6}$). This is indicated in figure 8 for half-full spherical tanks oscillated at an excitation-amplitude parameter $X_0/D = 0.01$ (ref. 3). The value of the viscosity parameter for a full-scale tank is generally much less than $10^{-6}$. If the model-tank size and the slosh liquid are chosen such that the viscosity parameter of the model is also less than $10^{-6}$, the sloshing characteristics (except for the liquid damping) in the full-scale and model tanks will be very similar, and the effect of liquid viscosity on the slosh-force parameter may be neglected. The

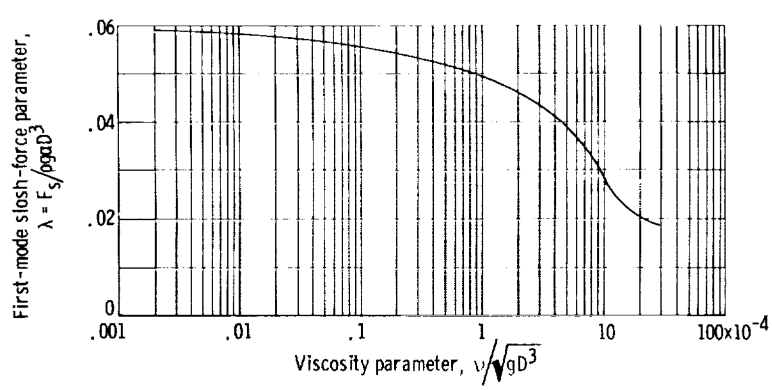



Figure 8. - First-mode slosh-force parameter as function of viscosity parameter for spherical tanks. Liquid-depth ratio, 0.5; excitation-amplitude parameter, 0.01.

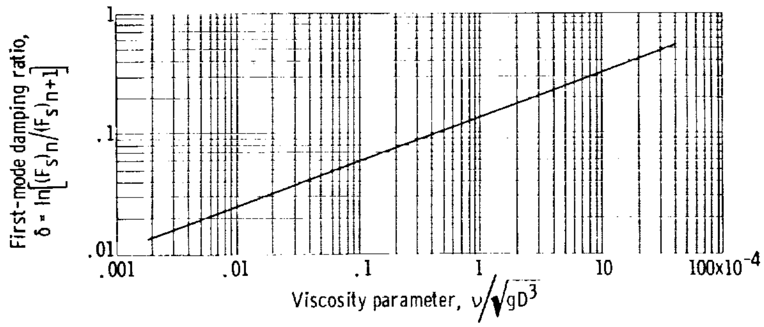



Figure 9. - First-mode damping ratio as function of viscosity parameter for spherical tanks. Liquid-depth ratio, 0.5.

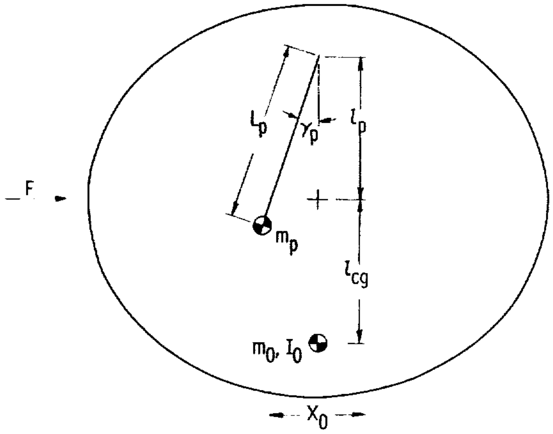



Figure 10. - Quantities used to describe pendulum analogy of liquid sloshing.

first-mode slosh forces expected to occur in geometrically similar tanks can be obtained from the values of the force parameter presented in figure 6 (where the viscosity parameter $\nu/\sqrt{gD^3} = 4.3\times10^{-7}$).

It is somewhat more difficult, however, to predict from scale-model-tank tests the first-mode damping ratios that are expected to occur in a full-scale tank when the values of the viscosity parameter for the two tanks are not nearly the same. In this case it is usually necessary to extrapolate experimental data obtained from several model-tank tests where the viscosity parameter has been varied over a substantial range. Figure 9 shows the viscous first-mode damping ratios obtained in half-full spherical tanks (ref. 3). In general, the damping decreases for any tank configuration (1) with a decrease in the liquid kinematic viscosity or (2) with an increase in the tank size or longitudinal acceleration imposed on the tank.

## Pendulum Analogy

The physical quantities necessary to represent the fundamental-mode liquid-sloshing characteristics as a pendulum analogy are shown in figure 10.

Pendulum (or sloshing) mass. - The effective pendulum mass that would produce oscillatory side forces equal to those that resulted from the sloshing liquid was experimentally determined by using equation (B6). The ratio of the sloshing mass to the total liquid mass present at a given liquid-depth ratio $m_p/m_t$ is shown in figure 11 for the oblate-spheroidal and spherical tank configurations. At low liquid-depth ratios, almost all of the liquid mass in each tank acted as the sloshing mass, while at the higher depth ratios, only a small portion of the liquid mass acted as the sloshing mass. The sloshing-mass ratio was greater for the oblate-spheroidal tank at a given depth ratio.

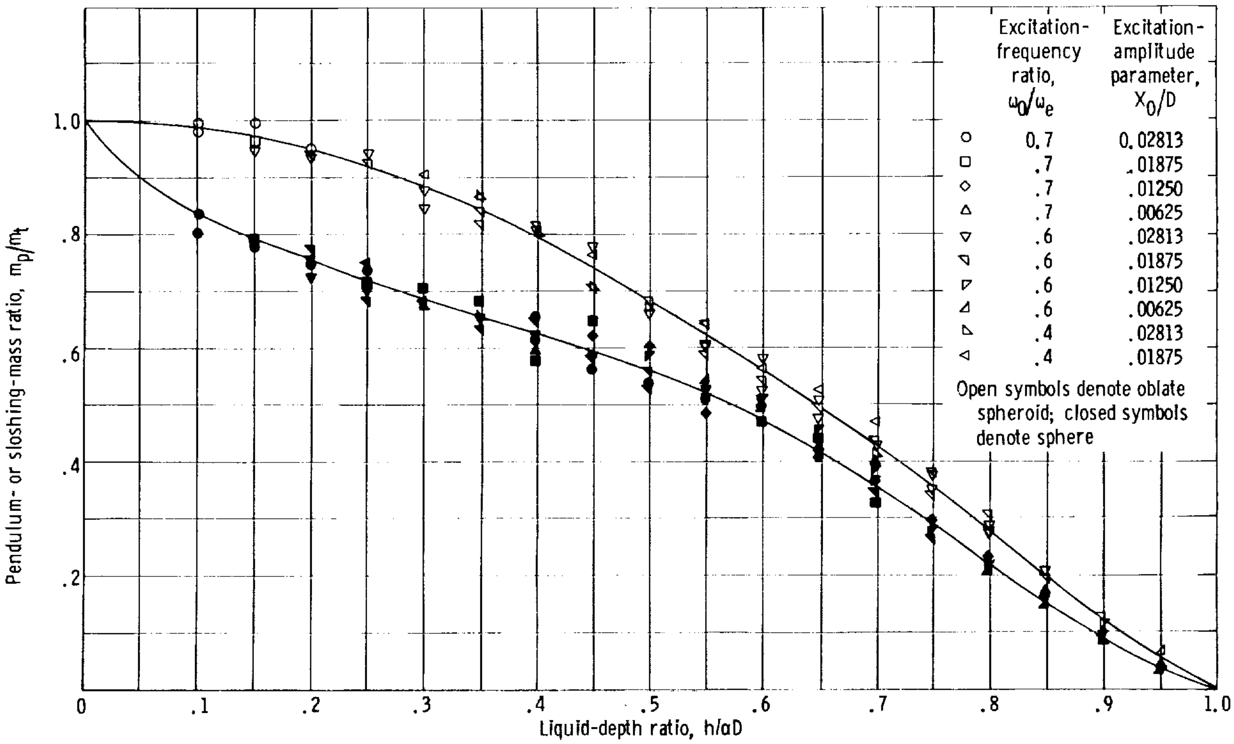



Figure 11. - Ratio of pendulum or sloshing mass to total liquid mass in partially filled tank.

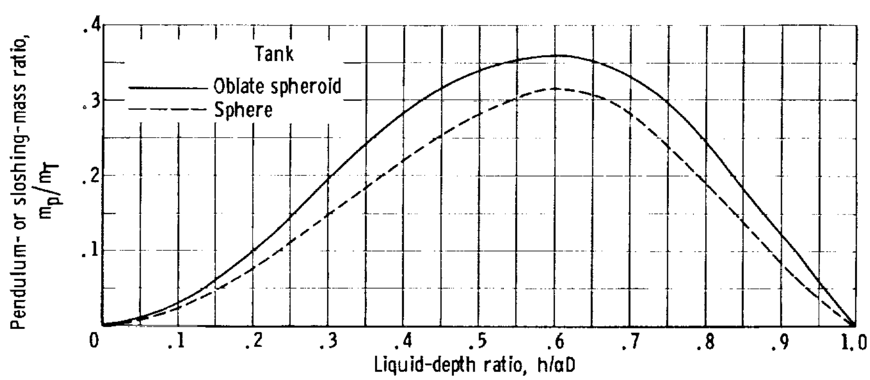



Figure 12. - Ratio of pendulum or sloshing mass to total liquid mass in completely filled tank.

No dependence of the sloshing-mass ratios on either the excitation frequency or the excitation amplitude was noted.

The sloshing mass ratio $m_p/m_T$ for a range of liquid depth ratios $0 \leq h/\alpha D \leq 1.0$ is presented in figure 12 where the sloshing or pendulum mass $m_p$ for a given liquid depth has been arbitrarily divided by a constant, namely the total liquid mass $m_T$ contained in a completely filled tank. The curves shown here represent values obtained from the faired curves presented in figure 11. The maximum sloshing mass was obtained at a liquid-depth ratio of 0.6 for both tank configurations, although the maximum slosh forces had been obtained at a depth ratio of 0.5 (fig. 6).

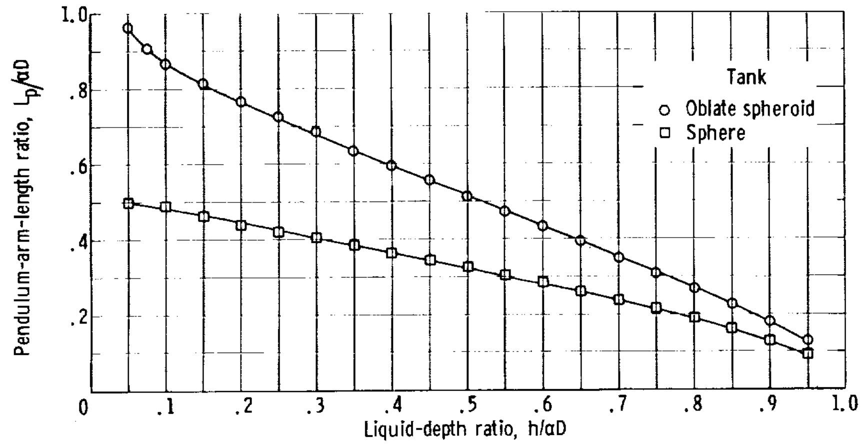



Figure 13. - Pendulum-arm-length ratio.

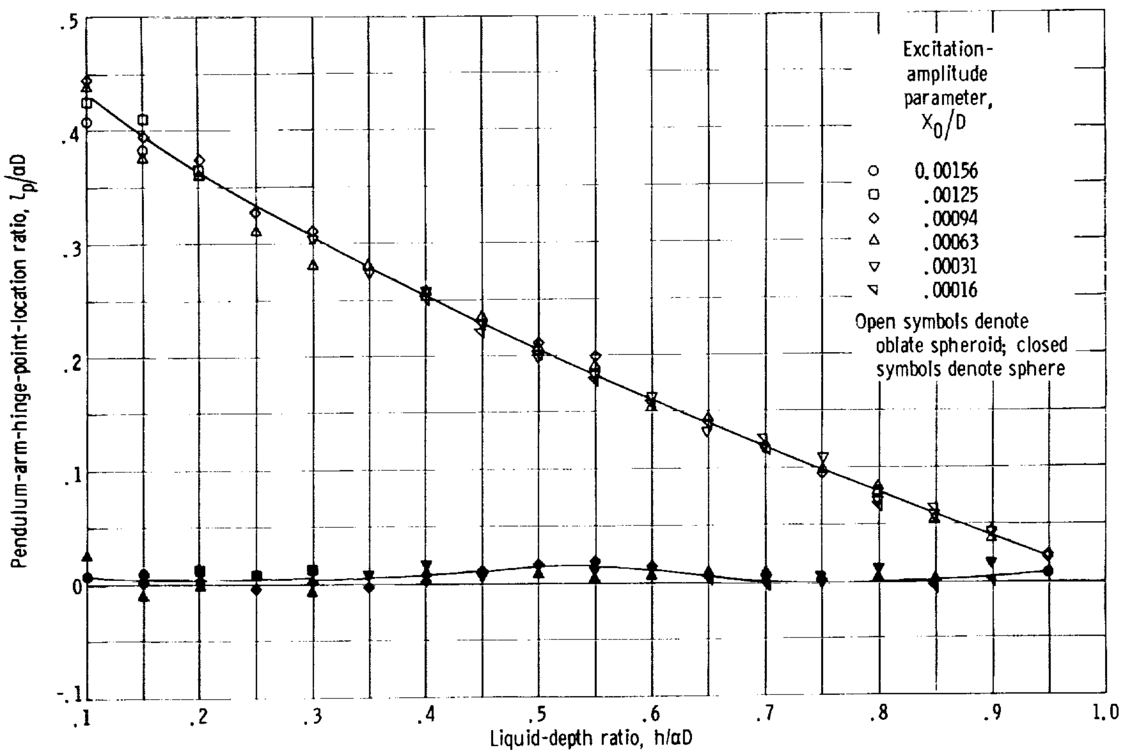



Figure 14. - Pendulum-arm-hinge-point-location ratio.

Pendulum arm length. - The pendulum arm length for each tank configuration was calculated directly from the pendulum fundamental-frequency expression $L_p = g/\omega_e^2$; the ratio of pendulum arm length to tank height $L_p/\alpha D$ is presented in figure 13. The pendulum-arm-length ratio decreased as the depth ratio increased and was smaller for the spherical tank at a given liquid-depth ratio.

Hinge-point location of pendulum arm. - The location of the hinge point of the pendulum arm was assumed to lie on the vertical axis of symmetry of the tank, and its location on that axis was determined by using equation (B9). The ratio of hinge-point location to tank height $\ell_p/\alpha D$ is presented in figure 14 for the two tank configurations. The hinge-point location moved from near the top to the center of the oblate-spheroidal tank as the liquid-depth ratio increased from 0 to 1.0. The hinge-point location tended to remain at the center of the spherical tank regardless of the liquid-depth ratio.

Pendulum angle. - The validity of equation (B6) (appendix B), from which the pendulum mass was determined, depended on the assumption that the pendulum arm remained normal to the liquid surface. The angles $\gamma_p$ from the vertical, through which the pendulum mass must oscillate to produce side forces equal to the first-mode horizontal slosh forces, were calculated from equation (B2) for a range of liquid-depth ratios $(0.2 \leq h/\alpha D \leq 0.8)$ and excitation amplitudes (0.010 to 0.200 in.) for both tank configurations. The slosh angles $\gamma_\ell$ were measured visually for the same initial sloshing conditions ($\omega_0$ and $X_0$) as for the calculated values of the pendulum angles $\gamma_p$. A comparison of the calculated ($\gamma_p$) and measured ($\gamma_\ell$) angles is shown in figure 15 for the oblate and spherical tanks. Agreement was generally good, and the assumption that the pendulum arm remained normal to the surface of the liquid (appendix B) was thereby confirmed.

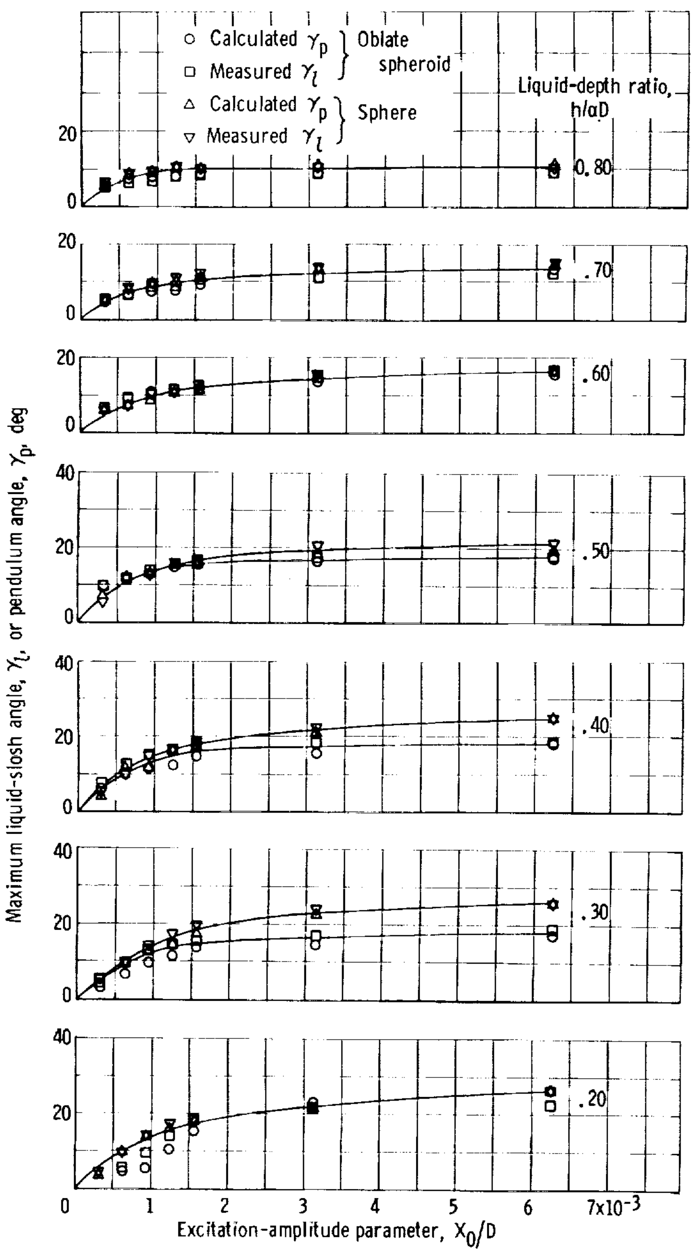


Figure 15. - Comparison of liquid-slosh angle and pendulum angle.

Fixed (or nonsloshing) mass. - The fixed mass was determined from equation (B10) and is presented in the form of the fixed-mass ratio $m_0/m_T$ in figure 16 for the two tank configurations. The fixed-mass ratio increased with an increase in liquid-depth ratio. The curves shown here represent values obtained from the faired curves presented in figure 11.

Scaling factors. - The length of the pendulum arm depends only upon the fundamental frequency of the contained liquid. Therefore, the pendulum-arm-length ratio for

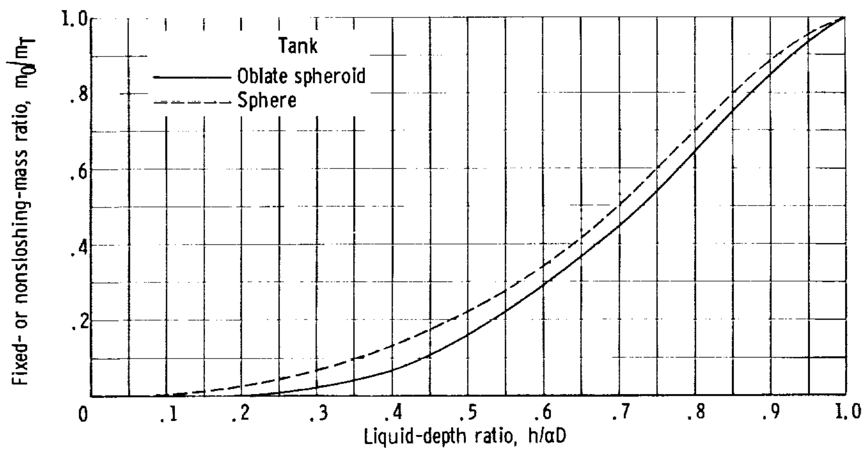



Figure 16. - Fixed- or nonsloshing-mass ratio.

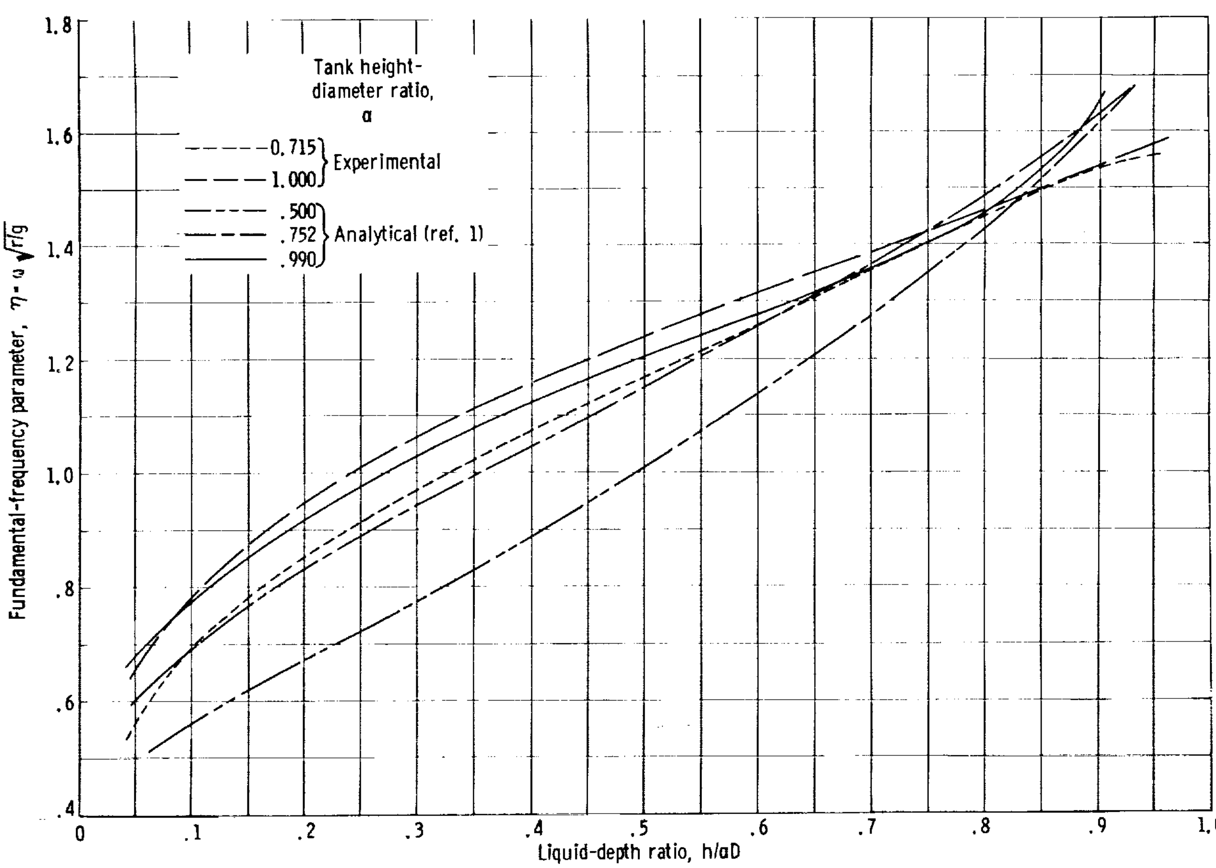



Figure 17. - Comparison of experimental fundamental-frequency parameter with analytical values.

geometrically similar tanks is independent of the tank size, the imposed longitudinal acceleration, and generally, the kinematic viscosity of the contained liquid.

The values of the pendulum and fixed-mass ratios, the hinge-point-location ratio, and the pendulum angles that are presented should be independent of the tank size and imposed longitudinal acceleration for geometrically similar tanks for values of the viscosity parameter less than approximately $10^{-6}$

## Comparison of Experimental Pendulum Analogy Parameters With Analytical Values

An analytical solution to the hydrodynamic equations of liquid motion and the equivalent pendulum-analogy parameters for oblate-spheroidal tanks of varying eccentricity (tank height-diameter ratio) were obtained by A. H. Hausrath. The results of this investigation are presented in reference 1 for tank height-diameter ratios $\alpha$ of 0.990, 0.752, and 0.500. A comparison of these analytical results with the experimental results obtained in this investigation for tank height-diameter ratios of 1.00 and 0.715 showed the following:

(1) The analytical and experimental values of the fundamental-frequency parameter (fig. 17) agreed fairly well except that (a) some difference in results was noted at liquid-depth ratios above 0.7 and (b) analytical values for $\alpha = 0.752$ appeared to be low for liquid-depth ratios less than 0.7.

(2) Analytical and experimental values of the pendulum-mass ratio $m_p/m_T$ (fig. 18) tended to reach a maximum at approximately the same liquid-depth ratio. However, the experimental values were as much as 56 percent greater at some depth ratios.

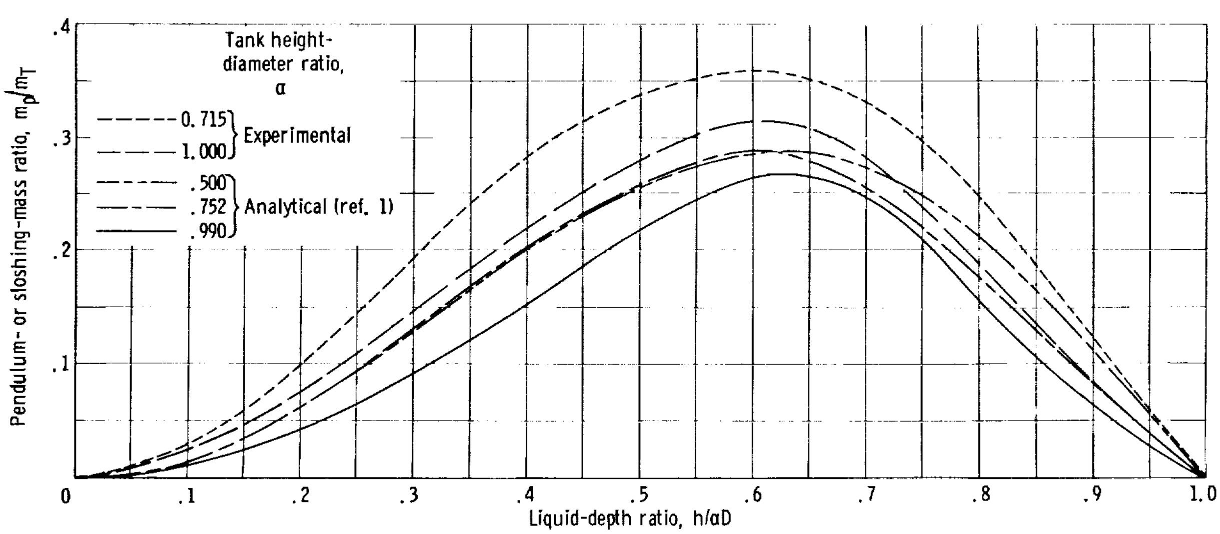



Figure 18. - Comparison of experimental pendulum- or sloshing-mass ratio with analytical values.

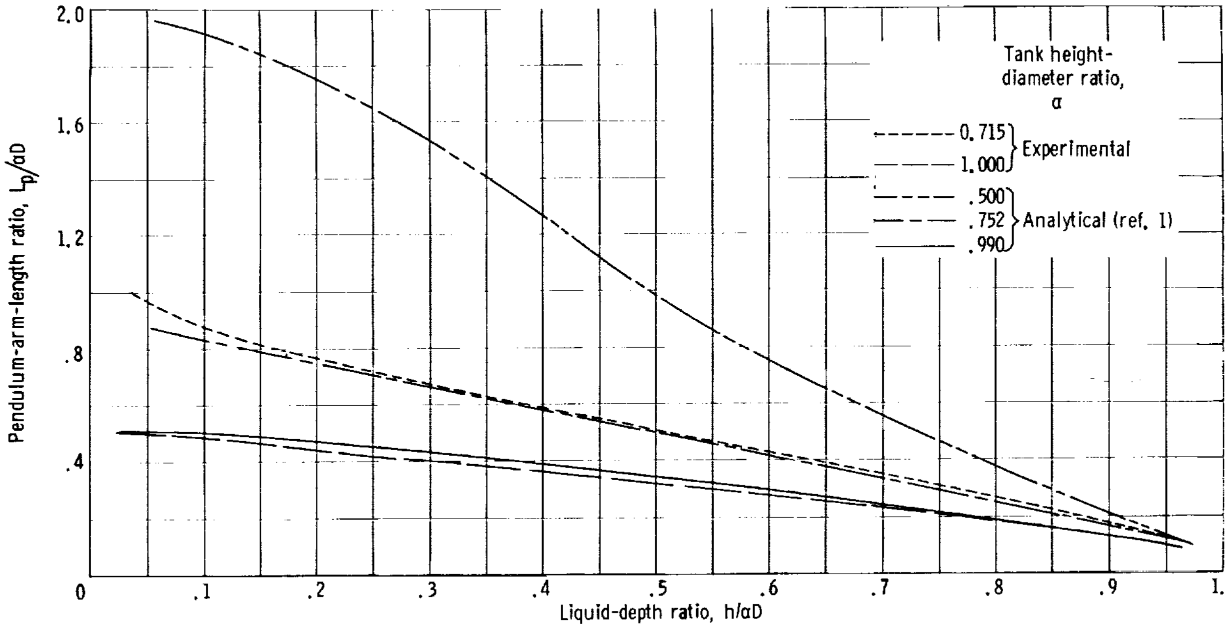



Figure 19. - Comparison of experimental pendulum-arm-length ratio with analytical values.

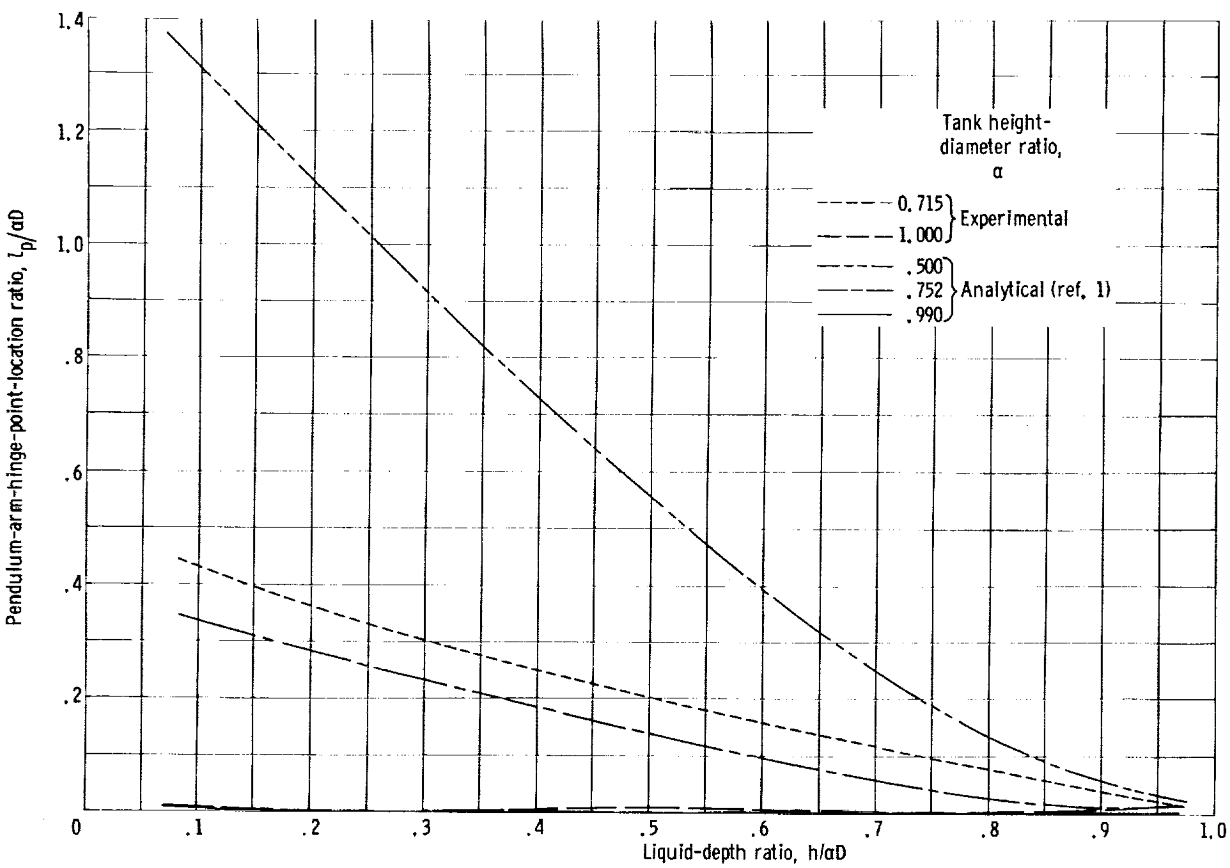



Figure 20. - Comparison of experimental pendulum-arm-hinge-point-location ratio with analytical values.

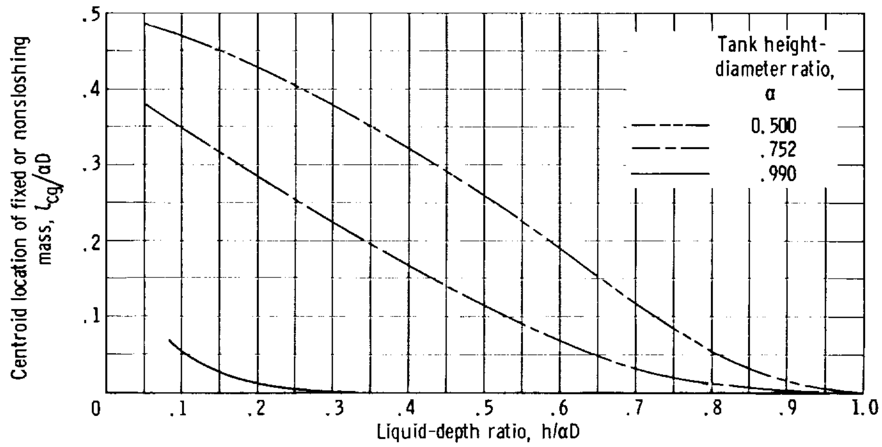



Figure 21. - Analytical values of centroid location of fixed or nonsloshing mass. (Data from ref. 1.)

(3) Analytical and experimental values of the pendulum-arm-length ratio (fig. 19) showed good agreement.

(4) Analytical and experimental values of the pendulum-arm-hinge-point-location ratio (fig. 20) showed good agreement.

Analytical values only of the centroid location of the fixed mass are presented in figure 21 since it was not possible to determine this quantity experimentally. The centroid location ratio $\ell_{cg}/\alpha D$ generally tended to decrease with an increase in (1) the tank height-diameter ratio $\alpha$ and (2) the liquid-depth ratio. The spherical-tank configuration would be an exception in that the centroid location of the fixed mass should remain at the center of the tank regardless of the liquid-depth ratio.

## SUMMARY OF RESULTS

An experimental investigation was conducted to determine the general liquid-sloshing characteristics and a pendulum analogy that would effectively represent the fundamental mode of liquid sloshing in oblate-spheroidal and spherical tanks over a range of liquid-depth ratios. The experimental results are presented in terms of dimensionless parameters and are compared, whenever possible, with analytical results. These results may be summarized as follows:

1. The fundamental-frequency parameters increased with an increase in the liquid-depth ratio. Values of the analytical, calculated, and experimental fundamental-frequency parameters were within 4 percent in all cases.

2. The maximum first-mode slosh-force parameters obtained for the oblate-spheroidal and spherical tanks were found when each tank was at or very near a half-full condition.

3. The first-mode damping ratios for low liquid-depth ratios increased with a decrease in the excitation-amplitude parameter or liquid-depth ratio. At high liquid-depth

ratios, the damping ratio increased with an increase in the excitation-amplitude parameter or liquid-depth ratio.

4. The ratio of pendulum or sloshing mass to total mass in the partially filled tank decreased as the liquid-depth ratio increased. The ratio of sloshing mass to total mass in the completely filled tank attained maximum values at a liquid-depth ratio of about 0.6. Values of both ratios were slightly greater for the oblate-spheroidal tank at a given liquid-depth ratio.

5. The ratio of pendulum arm length to tank height decreased as the liquid-depth ratio increased; values of this ratio were greater at a given liquid-depth ratio for the oblate-spheroidal tank.

6. The hinge-point location of the pendulum arm moved from near the top to the center of the oblate-spheroidal tank as the liquid-depth ratio increased, while the hinge-point location remained in the center of the spherical tank regardless of the liquid-depth ratio.

7. Close agreement between the calculated values of the pendulum angle and the measured slosh angles indicated that the pendulum arm remained normal to the liquid surface over a range of liquid-depth ratios for both tank configurations.

8. The ratio of fixed mass to total mass in the completely filled tank increased as the liquid-depth ratio increased.

9. Experimental values of the fundamental-frequency, pendulum-arm-length, and hinge-point-location parameters for both tank configurations showed good agreement with analytical values obtained previously. Experimental values of the ratio of sloshing mass to total mass in the completely filled tank showed the same characteristics but were somewhat greater than the analytical values. Analytical values of the fixed-mass centroid-location parameter generally tended to decrease with an increase in (1) the tank height-diameter ratio and (2) the liquid-depth ratio.

10. The values of the fundamental-frequency parameter and the pendulum-arm-length ratio are generally applicable to tanks (1) of any size having similar geometric configurations and (2) under any imposed longitudinal acceleration; the kinematic viscosity of the contained liquid generally should not have any effect. All the remaining liquid-sloshing and pendulum-analogy parameters presented, with the exception of the damping ratios, are believed to be applicable under conditions (1) and (2) as long as the value of the viscosity parameter is less than approximately $10^{-6}$. Appropriate values of damping in large-scale tanks must generally be extrapolated from small-scale tests in geometrically similar tanks for a varying viscosity parameter.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, November 6, 1964.

# APPENDIX A

## SYMBOLS

| | |
|---|---|
| $b$ | minor axis of oblate-spheroidal tank, ft |
| $D$ | circular diameter of tank, ft |
| $F$ | force on tank producing propellant sloshing, lb |
| $F_s$ | horizontal (side) slosh force, lb |
| $g$ | vertical acceleration of tank, 32.174 ft/sec$^2$ |
| $h$ | liquid depth, ft |
| $h_c$ | liquid depth in flat-bottom cylindrical tank of radius $r$ containing same liquid volume as partially filled oblate-spheroidal or spherical tank, $\frac{1}{3}\left[\frac{h(1.5\ \alpha D - h)}{\alpha D - h}\right]$, ft |
| $h/\alpha D$ | liquid-depth ratio |
| $I_0$ | moment of inertia of fixed (nonsloshing) mass, slugs/sq ft |
| $L_p$ | length of pendulum arm, ft |
| $L_p/\alpha D$ | pendulum-arm-length ratio |
| $\ell_{cg}$ | distance from center of tank to centroid of fixed mass, ft |
| $\ell_{cg,q}$ | distance from centroid of fixed mass to vehicle center of rotation |
| $\ell_{cg}/\alpha D$ | centroid location of fixed-mass ratio |
| $\ell_p$ | distance from center of tank to hinge point of pendulum arm, ft |
| $\ell_{p,q}$ | distance from hinge point of pendulum arm to vehicle center of rotation |
| $\ell_p/\alpha D$ | pendulum-arm-hinge-point-location ratio |
| $M$ | external moment on tank produced by liquid sloshing, ft-lb |
| $m_p$ | pendulum (effective liquid sloshing) mass, slugs |
| $m_T$ | total liquid mass present in completely filled tank, slugs |
| $m_t$ | total liquid mass present in partially filled tank, slugs |
| $m_0$ | fixed (effective liquid nonsloshing) mass, slugs |
| $q$ | vehicle center of rotation |
| $r$ | radius of liquid surface of partially filled tank, ft |
| $t$ | time, sec |
| $X_0$ | maximum excitation amplitude, ft |
| $X_0/D$ | excitation-amplitude parameter |

$x$ direction of force on tank producing propellant slosh

$x_0$ excitation amplitude at any time $t$, $X_0 \sin \omega_0 t$, ft

$\alpha$ ratio of tank height to diameter, b/D for oblate spheroid, D/D = 1 for sphere

$\gamma_\ell$ angle from horizontal through which liquid surface oscillates, radians

$\gamma_p$ angle from vertical through which pendulum oscillates, radians

$\delta$ first-mode damping ratio (logarithmic decrement), $\ln \left[(F_s)_n/(F_s)_{n+1}\right]$

$\epsilon$ first root of $J_1'(\epsilon_n) = 0$, 1.841

$\eta_a$ analytical fundamental-frequency parameter, $\omega_a \sqrt{r/g}$

$\eta_c$ fundamental-frequency parameter calculated from equivalent-cylindrical-tank method, $\omega_c \sqrt{r/g}$

$\eta_e$ experimental fundamental-frequency parameter, $\omega_e \sqrt{r/g}$

$\theta$ angular rotation about q, radians

$\lambda$ slosh-force parameter, $F_s/\rho g \alpha D^3$

$\nu$ liquid kinematic viscosity, sq ft/sec

$\nu/\sqrt{gD^3}$ viscosity parameter

$\xi$ damping factor, $\delta/2\pi$

$\rho$ liquid mass density, slugs/cu ft

$\sigma$ ratio of experimentally determined to calculated fundamental frequency (refs. 2 and 5)

$\omega$ fundamental frequency of liquid oscillation, radians/sec

$\omega_0$ excitation frequency of tank, radians/sec

Subscripts:

a analytically determined (ref. 4)

c calculated by equivalent cylindrical-tank method (refs. 2 and 5)

e experimentally determined

n cycle number, 1, 2, 3, . . .

Superscripts:

$\cdot$ first derivative with respect to time

$\cdot\cdot$ second derivative with respect to time

# APPENDIX B

## PRESENTATION OF LIQUID SLOSHING AS PENDULUM ANALOGY

Figure 22 shows a pendulum analogy of liquid sloshing for an arbitrary propellant-tank configuration symmetrical about its longitudinal axis and fixed in a rigid-body vehicle or test facility.

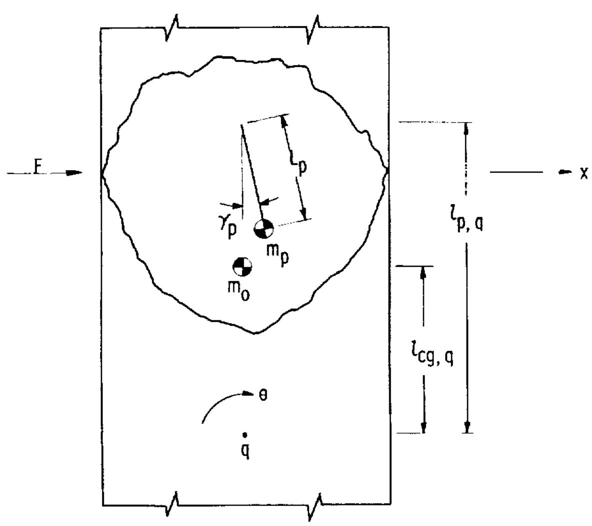



Figure 22. - Pendulum analogy of liquid sloshing.

The summation of forces acting on the tank in the x-direction that result in liquid sloshing may be written as

$$F = -m_0\ddot{x}_0 - m_0\ell_{cg,q}\ddot{\theta} + m_p g\gamma_p \quad \text{(B1)}$$

Small angle approximations have been assumed in all equations. In this investigation, the tank and test facility were not subjected to any rotational, or pitching, motion so that $\ddot{\theta} = 0$. In addition, the normal test procedure was to oscillate sinusoidally the tank and the contained liquid at a given excitation amplitude and frequency until the liquid surface had attained its maximum wave height. The motion of the tank was then quick-stopped, after which $\ddot{x}_0 = 0$, and the forces resulting only from the liquid sloshing were experimentally measured ($F = F_s$). Equation (B1) may then be written as

$$F_s = m_p g\gamma_p \quad \text{(B2)}$$

Equation (B2) would be useful in calculating the pendulum mass if the pendulum angle $\gamma_p$ corresponding to a specific slosh force $F_s$ were known. The angle $\gamma_p$ could be determined directly if the pendulum arm were assumed to remain normal to the liquid surface ($\gamma_p = \gamma_\ell$). It was difficult, however, to obtain sufficiently accurate measurements of the liquid-slosh angle $\gamma_\ell$ visually, and a second method of determining the pendulum mass was utilized.

The equation of the pendulum motion may be written as

$$m_p L_p \ddot{\gamma}_p + 2m_p\xi\omega_e L_p\dot{\gamma}_p + m_p L_p\omega_e^2\gamma_p = -m_p\ddot{x}_0 - m_p(\ell_{p,q} - L_p)\ddot{\theta} \quad \text{(B3)}$$

If the damping factor $\xi$ is small, the damping term $(2m_p \xi \omega_e L_p \dot{\gamma}_p)$ can be considered negligible, and since $\ddot{\theta} = 0$ for this investigation, equation (B3) may be rewritten as

$$m_p L_p(\ddot{\gamma}_p + \omega_e^2 \gamma_p) = -m_p \ddot{x}_0 \tag{B4}$$

When the liquid mass was oscillated at an excitation frequency much lower than its fundamental frequency ($\omega_0 << \omega_e$), the liquid oscillated at exactly the excitation frequency as long as the oscillatory motion of the tank continued. Since $\gamma_p$ was assumed to be equal to $\gamma_\ell$, it can also be assumed that $\ddot{\gamma}_p = \ddot{\gamma}_\ell = -\omega_0^2 \gamma_\ell \sin \omega_0 t$. At the maximum wave height of the liquid surface, then, it is assumed that $\ddot{\gamma}_p = -\omega_0^2 \gamma_p$. By making this substitution, equation (B4) becomes

$$m_p L_p \gamma_p(-\omega_0^2 + \omega_e^2) = -m_p \ddot{x}_0 \tag{B5}$$

Equation (B5) must again be rewritten in terms of measurable quantities by making the following substitutions:

(1) $L_p = g/\omega_e^2$ (pendulum frequency expression)

(2) $\ddot{x}_0 = -\omega_0^2 X_0$ (at maximum wave height of liquid surface)

(3) $m_p = F_s/g\gamma_p$ (substitute into left side only of eq. (B5))

The resulting equation is then

$$m_p = \frac{F_s}{X_0}\left[\frac{1}{\omega_0^2} - \frac{1}{\omega_e^2}\right] \tag{B6}$$

Equation (B6) is valid for determining the pendulum or effective-sloshing mass as long as the damping factor $\xi$ is small and, therefore, the damping term in equation (B3) is negligible. Consequently, the values of the first force peaks occurring immediately after the quick stop are almost the same values as the steady-state slosh forces that occurred while the tank was subjected to the oscillatory motion.

Once the pendulum mass has been determined for a given liquid-depth ratio, the pendulum angle $\gamma_p$ can be calculated for any corresponding value of the horizontal slosh force by using equation (B2).

The hinge-point location of the pendulum arm may be determined from a summation of moments about the vehicle center of rotation q:

$$M = -m_0 \ell_{cg,q} \ddot{x}_0 - (I_0 + m_0 \ell_{cg,q})\ddot{\theta} + m_p \ell_{p,q} g \gamma_p \tag{B7}$$

The test facility was not subjected to any rotational, or pitching, motion ($\ddot{\theta} = 0$), however, and a summation of moments about the center of the tank may be written as

$$M = -m_0 \ell_{cg} \ddot{x}_0 + m_p \ell_p g \gamma_p \tag{B8}$$

Since $m_p = F_s/g\gamma_p$ and $\ddot{x}_0 = 0$ after the tank has been quick-stopped, the location of the hinge point above the center and on the longitudinal axis of the tank may be determined from

$$\ell_p = M/F_s \tag{B9}$$

No feasible method was available to determine experimentally the fixed or effective nonsloshing mass or its effective centroid location. It may be assumed, however, that the fixed mass may be determined from the following equation:

$$m_0 = m_t - m_p \tag{B10}$$

The ability of a pendulum analogy to represent the fundamental mode of liquid sloshing in a cylindrical tank is discussed in reference 6 for two cases: (1) translational oscillatory tank motion, and (2) pitching oscillatory tank motion. Additional information on the numerical values of the pendulum analogy parameters for spherical and cylindrical tanks, as well as the effect of liquid sloshing upon vehicle stability, can be obtained from other sources (refs. 7 and 8) as well as from reference 1. It should be noted, however, that some differences in numerical values for the fundamental frequency parameter, the sloshing or pendulum mass, and the effective centroid location of the fixed mass were found to exist between the experimental and/or analytical results presented herein and those presented in reference 8.

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